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TRANSLATION

TRANSIENTS IN THE BURNING OF POWDERS

By

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FOREIGN TECHNOLOGY DIVISION

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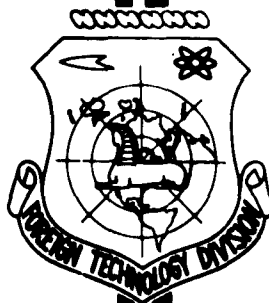
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English Pages: 12

Source: Russian Periodical, Zhurnal
Prikladnoy Mekhaniki i
Tekhnicheskoy Fiziki, No. 5,
1962, pp. 83-88.

T-11
SOV/207-62-0-5

TRANSIENTS IN THE BURNING OF POWDERS

B. V. Novozhilov

(Moscow)

During the steady-state burning of powder, the burning velocity
and temperature distribution are constant, i.e., they do not depend
upon time. If in some manner the steady state is disrupted (e.g.,
change of pressure), these magnitudes will begin to change with time
and the burning will become nonsteady-state in which case, eventually,
a steady-state regime corresponding to the new conditions is established.
Such a transient from one steady-state regime to another can not be
accomplished instantaneously because of the finite nature of the
relaxation times of the different processes which determine the burn-
ing of powder. In conjunction with this there emerges the question
of nonsteady-state burning of powder during a transient. The question
of nonsteady-state burning of powders was first examined by Ya. B.
Zel'dovich [1].

Here we examine the simplest nonsteady-state processes during the
transition of the burning of powder from one steady-state regime to
another. By numerically solving equations, we determine the time-
dependences of the burning velocity, pressures in the chamber, and

temperature profiles in the solid phase.

1. Statement of the Problem

Let us examine two types of transients:

1) the transition from a steady-state regime of burning at initial pressure p_0, to a steady-state regime of burning at pressure p_1, during which pressure changes instantaneously;

2) the transient in a powder chamber from steady-state burning at pressure p_0 to steady-state burning at pressure p_1. In this case the transition is accomplished by an instantaneous change of the nozzle section from the value σ_0, corresponding to p_0, to the value σ_1, corresponding to p_1.

First of all let us start with the question of the relaxation times of various processes during the burning of powder in a chamber. If we examine the plane case (Fig. 1), the entire region $-\infty < x < \infty$ can be divided into three parts (the coordinate system is associated with the solid phase):

Region (1) — Solid phase $0 < x < \infty$, where only heating of the powder takes place. In this region chemical reactions do not take place.

Region (2) — combustion products $(-\infty < x < b)$.

Region (3) $(b < x < 0)$. Here chemical reactions take place, in the solid phase $(d < x < 0)$, as well as in the gaseous phase $(b < x < d)$. In this way, all of the chemical reactions take place in region (3), where $x = 0$ corresponds to the beginning of the reaction in the solid phase, and $x = b$ corresponds to the termination of the reaction in the gaseous phase.

The relaxation time of the distribution of temperatures in

preheating region (1) is determined by the linear burning velocity u and by the thermal diffusivity of the powder ϰ; it equals

$$t_1 = \varkappa/u^2 \qquad (1.1)$$

In region (2) we must examine two time characteristics; first, the time of the pressure change due to gas discharge from the nozzle and, second, the time for pressure equalization throughout the chamber. The characteristic discharge time

$$t_2 \quad \frac{V}{a \sigma A} \quad \frac{V p}{a \rho u S} \qquad (1.2)$$

where V is the free volume of the chamber, a is the powder strength, σ is the critical nozzle section, A is the discharge coefficient, p is pressure, ρ is the density of the powder, and S is the burning surface.

The time of pressure equalization throughout the chamber will be

$$t_3 = l/c_0 \qquad (1.3)$$

where l is the characteristic size of the chamber, and c_0 is the speed of sound.

Three time characteristics can be introduced in region (3): the reaction time in the condensed phase, the time of the preheating of the gas up to the reaction temperature in the gaseous phase, and the reaction time in the gaseous phase.

The reaction time in the condensed phase can be defined as the ratio of the thickness of the reaction layer d to the burning velocity. On the other hand, the order of magnitude of d is

$$d \quad \Delta T \left/ \left| \frac{dT}{dx} \right|_{x=0} \right.$$

where ΔT is the temperature interval (of the order of several degrees),

in which the chemical reaction takes place. Under steady-state conditions

$$\left|\frac{dT}{dx}\right|_{x=0} = (T_1 - T_0)\frac{u}{\varkappa}$$

where T_1 is the temperature when $x = 0$, and T_0 is the initial temperature of the powder (when $x = \infty$). Finally we have

$$t_1 \sim \frac{\Delta T}{T_1 - T_0}\frac{\varkappa}{u^2} \sim \frac{\Delta T}{T_1 - T_0}t_1 \qquad (1.4)$$

The time of preheating of gases up to the burning temperature equals

$$t_5 \sim \frac{D\rho_1^2}{u^2\rho^2} \qquad (1.5)$$

where D is the coefficient of diffusion, and ρ_1 is the density of the gas.

Finally, for the reaction times in the gaseous phase we get an estimate analagous to (1.4), i.e.,

$$t_6 \sim \frac{\Delta T'}{T_2 - T_1}t_5 \qquad (1.6)$$

where $\Delta T'$ is the temperature interval, in which the chemical reaction takes place, and T_2 is the combustion temperature.

From these estimates it is obvious that the times t_4 and t_6 are much smaller than times t_1 and t_5, respectively. Thus, times t_4 and t_6 can be immediately disregarded in an examination of nonsteady-state processes. Let us represent the values of the times in seconds.

$$t_1 = 6\cdot10^{-1}, \qquad t_2 = 10^{-3}, \qquad t_3 = 10^{-1}, \qquad t_5 = 5\cdot10^{-6} \text{ when } p = 1 \text{ atm}$$
$$t_1 = 10^{-3}, \qquad t_3 = 5\cdot10^{-4}, \qquad t_4 = 10^{-4}, \qquad t_5 = 10^{-5} \text{ when } p = 100 \text{ atm}$$

Here we assumed $u[\text{cm/sec}] = 0.04p^{2/3}$ [atm]

$$a = 8\cdot10^3 \text{ atm cm}^3/g, \qquad \varkappa = 9\cdot10^{-4}\frac{cm^2}{sec}, \qquad D = 1\frac{cm^2}{sec}$$
$$\rho = 1.6 \text{ g/cm}^3, \qquad V = 100 \text{ cm}^3, \qquad S = 200 \text{ cm}^2, \qquad l = 10 \text{ cm } \& c_0 = 10 \text{ cm/sec}.$$

From these data it is obvious that relaxation times t_3 and t_5 can

be disregarded compared with t_1 and t_2. Therefore we can consider that region (3) relaxes much more rapidly than regions (1) and (2). Henceforth we will examine only the relaxation times of regions (1) and (2), considering that the change in values in region (3) with time are instantaneous.

Moreover, we will consider that the pressure and temperature of the combustion products are constant throughout the chamber. Temperature T_1 will also be considered constant, since it is known that the change in temperature on the surface of the powder, as a function of pressure, is slight compared with the temperature itself.

2. Basic Equations

The burning velocity of powder in a steady-state regime depends upon the pressure p and initial powder temperature T_0. We will consider that the burning velocity of the powder is determined by the formula

$$u = Bp^{\nu}(1 + aT_0) \qquad (B,\ a \ \text{const},\ \nu = {}^{2}/_{3}) \qquad (2.1)$$

The temperature profile in the solid phase is given by the well-known Michaelson solution

$$T(x) = T_0 + (T_1 - T_0)\exp\left(\frac{ux}{\chi}\right) \qquad (2.2)$$

In the nonsteady-state regime, the burning velocity and temperature profile will not be determined by derived expressions. The burning velocity will be determined by the rate of the chemical reactions in region (3), which depend greatly upon temperature; this temperature depends not directly on the temperature when $x = \infty$, but on the temperature gradient when $x = 0$. Therefore for nonsteady-state burning we should use not the relation $u = u(p,T_0)$, but the relation $u(p,\varphi)$,

where $\varphi = (\partial T/\partial x)_{x=0}$.

For the steady-state case, from (2.1) and (2.2) we get

$$u = Bp^\nu \frac{(1+\alpha T_1)}{2}\left[1 + \left(1 + \frac{4\kappa x}{(1+\alpha T_1)^2 Bp^\nu}\left(\frac{dT}{dx}\right)_{x=0}\right)^{1/2}\right]$$

In connection with the above, the preceding formula, can be immediately generalized for the nonsteady-state case, where instead of $(dT/dx)_{x=0}$, we must substitute $(\partial T/\partial x)_{x=0} = \varphi$. Thus,

$$u = Bp^\nu \frac{(1+\alpha T_1)}{2}\left[1 + \sqrt{1 + \frac{4\kappa x}{(1+\alpha T_1)^2 Bp^\nu}\varphi}\right] \tag{2.3}$$

The temperature gradient in the solid phase should be determined from the equation of heat conductivity in region (1)

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x} + uT\right) \tag{2.4}$$

The pressure in the chamber is subject to the balance equation

$$\frac{dp}{dt} = \frac{u}{V}(\rho u S - p A \sigma) \tag{2.5}$$

The first term on the right describes the pressure increase in the chamber due to the burning of powder, the second term describes the decrease due to discharge.

A transient begins from a state of steady-state burning

$$t = 0, \qquad p = p_0, \qquad u = u_0 = Bp_0^\nu(1+\alpha T_0), \tag{2.6}$$
$$T(x, 0) = T_0 + (T_1 - T_0)\exp\left(-\frac{u_0 x}{\kappa}\right)$$

The boundary conditions have the form

$$T(0, t) = T_1, \qquad T(\infty, t) = T_0 \tag{2.7}$$

Let us now introduce the dimensionless magnitudes

$$\pi = \frac{p}{p_0}, \qquad \omega = \frac{u}{u_0}, \qquad \theta = \frac{T - T_0}{T_1 - T_0}, \qquad \xi = x\frac{u_0}{\kappa}, \qquad \tau = t\frac{u_0^2}{\kappa} \tag{2.8}$$
$$\gamma = \frac{V u_0/\kappa}{p_0 A S \kappa}, \qquad \beta = \frac{A \sigma p_0}{\rho S u_0}, \qquad \eta = 2\frac{1+\alpha T_0}{1+\alpha T_1}$$

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The dimensionless coordinate ξ and time τ are the ratio of the length and time to the characteristic length $\varkappa/u_0$ and time $\varkappa/u_0^2$ in the solid phase; the parameter γ is the ratio of the discharge time to the relaxation time of the solid phase at the initial pressure; β is the ratio of the mass velocity of gas discharge at pressure p_0 and profile σ to the mass of the gas which is released per unit time by the entire surface of the powder in a steady-state regime at the same pressure; the parameter η characterizes the degree of heating of the powder; the value $\eta = 2$ corresponds to a thoroughly heated powder. We can show that $\eta = 1$ corresponds to the limit of stable burning of the powder.

In dimensionless variables, Eqs. (2.3), (2.4), and (2.5) take the form

$$\omega = \frac{\tau'}{\eta}\left[1 + \left(1 - (2 + \eta^2)\frac{1}{\tau^2}\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=0}\right)^{1/2}\right] \tag{2.9}$$

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\xi^2} + \omega\theta \tag{2.10}$$

$$\gamma\frac{d\tau}{d\tau} = \omega - \beta\pi \tag{2.11}$$

with the initial conditions

$$\pi = 1, \quad \omega = 1, \quad \theta = e^{-\xi} \quad \text{when} \quad \tau = 0 \tag{2.12}$$

$$\theta(0,\tau) = 1, \quad \theta(\infty,\tau) = 0 \tag{2.13}$$

The initial state corresponds to the steady-state solution of these equations when $\beta = 1$.

The equations were solved using an electronic computer.

3. Transients With an Abrupt Change in Pressure Difference

The simplest nonsteady-state process is the transition from steady-state burning, when $p = p_0$, to a steady-state regime when $p = p_1$,

accomplished by an abrupt change in pressure from p_0 to p_1. In this case, the pressure during the nonsteady-state process remains constant (equal to p_1), and to solve the problem we need examine only the two equations (2.9) and (2.10).

In the first moment of time, the temperature profile in the solid phase corresponds to steady-state burning when $p = p_0$, and the pressure equals p_1. Therefore the burning velocity when $\tau = 0$ (the beginning of the transient) assumes the value

$$\omega\ (0) = \frac{\pi_1^v}{\eta}\left[1 + \left(1 - (2\eta - \eta^2)\frac{1}{\pi_1^v}\right)^{\prime\prime}\right] \tag{3.1}$$

When $\pi_1 > 1$ we have $\omega'(0) > \omega_0$; therefore the absolute value of the gradient at the surface should increase, which, in turn, leads to a decrease in the burning velocity. In time, the velocity tends toward $\omega_1 = \pi_1$, while the temperature distribution tends toward the Michaelson profile θ ($\xi, \infty = \exp(-\omega_1\xi)$), which corresponds to steady-state burning when $\pi = \pi_1$.

In Fig. 2, curves 1-4 give the dependences of the burning velocity ω on the time τ for the values $\pi_1 = 2$, 20, 50, and 200, respectively, for $\eta = 1.15$. Figure 3 gives the change of the temperature profile with time for $\pi_1 = 10$ and $\eta = 1.15$. Curves 1-6 correspond to the values $\tau = 0$, 0.016, 0.051, 0.121, 0.237, and ∞. Obviously, the greater π_1, the greater the velocity value $\omega\ (0)/\omega_1$, and the more rapid is the transition to a new steady-state regime (with a greater burning velocity the heated layer of powder burns more rapidly).

Figure 4 illustrates the dependence the nonsteady-state burning velocity on the degree of heating of the powder for $\pi_1 = 10$. Curves 1, 2, and 3, correspond to the values $\eta = 1.25$, 1.15, 1.05. The more the powder is heated (the greater is η), the less is the transient expressed. For a thoroughly heated powder ($\eta = 2$ and $T_0 = T_1$),

nonsteady-state phenomena are not observed and the velocity always satisfies the condition $\omega = \pi^{\nu}$.

In a nonsteady-state region an effective pressure exponent ν' can be introduced in the law of burning, which is important for the question of the stability of the burning of powder. In the steady-state case

$$\frac{u_1}{u_0} = \left(\frac{p_1}{p_0}\right)^{\nu} \qquad (3.2)$$

In the nonsteady-state case let us write, analogously,

$$\frac{u(t)}{u_0} = \left(\frac{p_1}{p_0}\right)^{\nu'} \qquad (3.3)$$

From this

$$\nu'(\tau) = \nu + \frac{\ln \omega(\tau) \cdot u_0}{\ln \pi_1} \qquad (3.4)$$

Figure 5 gives the dependences of the effective pressure exponent ν' on the law of burning (τ) when $\eta = 1.15$. Curves 1-4 correspond to the values $\tau = 2$, 10, 50, and 200. At the initial moment of time

$$\nu'(0) = \nu + \ln \frac{1}{\eta}\left[1 + \left(1 - (2\eta - \eta^2)\frac{1}{\pi_1^{\nu}}\right)^{1/2}\right] \Big/ \ln \pi_1 \qquad (3.5)$$

With decreasing π_1, the function $\nu'(0)$ increases and tends toward the value

$$\nu_0'(0) = \nu + \frac{2 - \eta}{2(\eta - 1)}\nu \qquad \pi_1 \to 1 \qquad (3.6)$$

4. **Transients During an Abrupt Change of the Nozzle Section**

During the steady-state burning of powder in a chamber, each nozzle section has a corresponding pressure, determined from (2.5) when $dp/dt = 0$

$$p = \frac{\omega u S}{A_3} \qquad (4.1)$$

i.e., $\beta = 1$. If the nozzle section is changed in a period of time which is short compared with the characteristic times t_1 and t_2, there

begins the nonsteady-state process of a transition from steady-state burning at p_0 to steady-state burning at p_1 which corresponds to the new value of the nozzle section.

With an abrupt decrease of the section, the velocity at the initial moment of time is not discontinuous, since the pressure in the chamber is equal to the initial pressure p_0. Then the pressure begins to increase, since with a smaller nozzle section the discharge takes place more slowly and $dp/dt > 0$.

The pressure increase brings about a velocity increase and a change of the temperature profile in the solid phase. With different values of the parameters γ and β (or π_1), the nature of the pressure change as a function of time will differ. The greater the γ (for the same β), i.e., the larger the ratio of the discharge time to the relaxation time of the solid phase, the less the nonsteady-state process differs from the quasisteady-state process in which the velocity is continually proportional to p^ν.

Figure 6 shows the dependence of pressure π on time τ for $\pi_1 = 100$ and $\eta = 1.15$. Curves 1-5 correspond to the values $\gamma = 0.001$, 0.002, 0.005, 0.008 and 0.01. Figure 7 shows the dependence of pressure and velocity on time when $\pi_1 = 50$, $\eta = 1.5$, and $\gamma = 0.002$. Figure 8 gives the change of temperature profile with time when $\pi_1 = 50$, $\eta = 1.15$, and $\gamma = 0.002$. Curves 1, 2, and 3 correspond to the values $\pi = 0$, 0.077 and ∞. From the graphs it is obvious that with low values of γ, the pressure in the chamber can exceed the magnitude of the absolute pressure p_1. Figure 9 presents the values of the relative pressure excesses $\Delta = \pi_* - \pi_1/\pi_1$ as a function of γ when $\eta = 1.15$, where π is the maximum value of the pressure in the chamber. Curves 1-5 correspond to the values $\pi_1 = 5$, 10, 50, 100, and 200. Figure

10 gives the time τ_* during which the pressure reaches its maximum vs. the parameter γ when $\eta = 1.15$. Curves 1-4 correspond to the values $\tau_1 = 5$, 10, 50, and 100.

The transients examined are the simplest ones, while the results obtained are accurate for times which exceed the relaxation times t_3, t_4, t_5, and t_6. In order to get correct results for short times, it is necessary to examine a more complex system than (2.9)-(2.11).

In conclusion, I wish to thank Ya. B. Zel'dovich for discussing the formulation of the problem and O. I. Leypunskiy for his help, and also A. A. Milyutin, G. G. Vilenskaya, and Ye. A. Prozer who solved the equations and carried out the calculations on the computer.
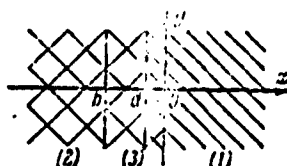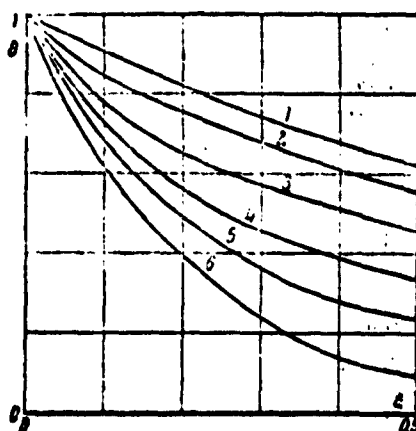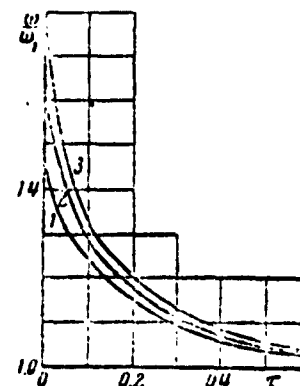
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Fig. 2.

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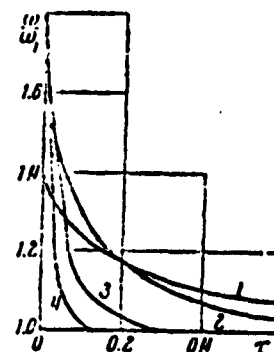
Fig. 4.

Fig. 5.

Fig. 6.

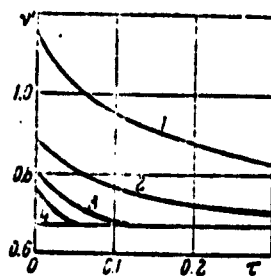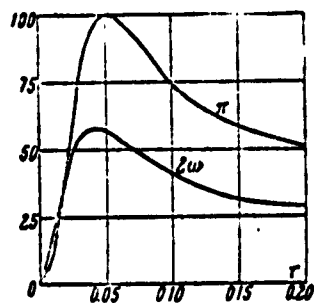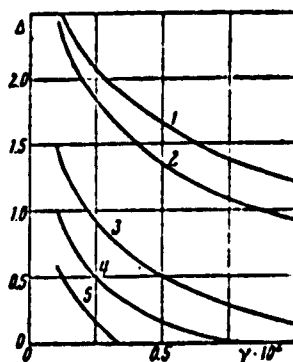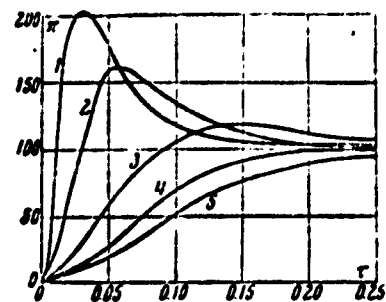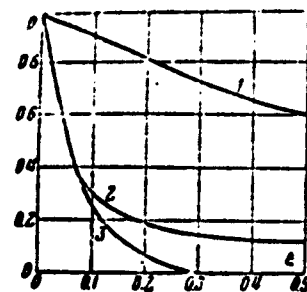Fig. 7.

Fig. 8.

Fig. 9

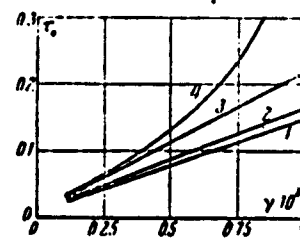Fig. 10.

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